and:

$$d_{12}^{(e)} = 2 \qquad d_{13}^{(e)} = \sqrt{2+3-2\sqrt{6}\tfrac{2}{\sqrt{6}}} = 1 \quad d_{14}^{(e)} = \sqrt{2+3+2\sqrt{6}\tfrac{2}{\sqrt{6}}} = 3$$
$$d_{23}^{(e)} = \sqrt{2+3} = \sqrt{5} \qquad d_{24}^{(e)} = \sqrt{5}$$
$$d_{34}^{(e)} = \sqrt{3+3+2*3\tfrac{1}{3}} = 2\sqrt{2}$$

Problem 4.9 :

The energy of the signal waveform $s'_m(t)$ is :

$$\begin{aligned}
\mathcal{E}' &= \int_{-\infty}^{\infty} |s'_m(t)|^2\, dt = \int_{-\infty}^{\infty} \left| s_m(t) - \frac{1}{M}\sum_{k=1}^{M} s_k(t) \right|^2 dt \\
&= \int_{-\infty}^{\infty} s_m^2(t)dt + \frac{1}{M^2}\sum_{k=1}^{M}\sum_{l=1}^{M}\int_{-\infty}^{\infty} s_k(t)s_l(t)dt \\
&\quad -\frac{1}{M}\sum_{k=1}^{M}\int_{-\infty}^{\infty} s_m(t)s_k(t)dt - \frac{1}{M}\sum_{l=1}^{M}\int_{-\infty}^{\infty} s_m(t)s_l(t)dt \\
&= \mathcal{E} + \frac{1}{M^2}\sum_{k=1}^{M}\sum_{l=1}^{M}\mathcal{E}\delta_{kl} - \frac{2}{M}\mathcal{E} \\
&= \mathcal{E} + \frac{1}{M}\mathcal{E} - \frac{2}{M}\mathcal{E} = \left(\frac{M-1}{M}\right)\mathcal{E}
\end{aligned}$$

The correlation coefficient is given by :

$$\begin{aligned}
\rho_{mn} &= \frac{1}{\mathcal{E}'}\int_{-\infty}^{\infty} s'_m(t)s'_n(t)dt = \frac{1}{\mathcal{E}'}\int_{-\infty}^{\infty}\left(s_m(t) - \frac{1}{M}\sum_{k=1}^{M} s_k(t)\right)\left(s_n(t) - \frac{1}{M}\sum_{l=1}^{M} s_l(t)\right)dt \\
&= \frac{1}{\mathcal{E}'}\left(\int_{-\infty}^{\infty} s_m(t)s_n(t)dt + \frac{1}{M^2}\sum_{k=1}^{M}\sum_{l=1}^{M}\int_{-\infty}^{\infty} s_k(t)s_l(t)dt\right) \\
&\quad -\frac{1}{\mathcal{E}'}\left(\frac{1}{M}\sum_{k=1}^{M}\int_{-\infty}^{\infty} s_n(t)s_k(t)dt + \frac{1}{M}\sum_{l=1}^{M}\int_{-\infty}^{\infty} s_m(t)s_l(t)dt\right) \\
&= \frac{\frac{1}{M^2}M\mathcal{E} - \frac{1}{M}\mathcal{E} - \frac{1}{M}\mathcal{E}}{\frac{M-1}{M}\mathcal{E}} = -\frac{1}{M-1}
\end{aligned}$$

Problem 4.10 :

(a) To show that the waveforms $f_n(t)$, $n = 1, \ldots, 3$ are orthogonal we have to prove that:

$$\int_{-\infty}^{\infty} f_m(t)f_n(t)dt = 0, \qquad m \neq n$$